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**BOUNDED INFLUENCE ESTIMATION FOR AUTO-
REGRESSIVE CONDITIONAL HETEROSKEDASTICITY
MODELS WITH ESTIMATES OF THE VARIANCES
OF FOREIGN EXCHANGE RATES**

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Abstract

This paper proposes a robust estimation procedure, the bounded influence estimate (BIE), that is robust against departure from the conditional normality of the autoregressive conditional heteroskedasticity (ARCH) models to describe the behavior of exchange rates. First, the BIE identifies the additive outliers (AO, e.g., Fox 1972) caused by abnormal information arrivals which may be triggered by changes in domestic policies and international shocks. Identification of outliers allows us to analyze the major economic and political factors that contribute directly to the dramatic changes in exchange rates. Second, the performance of the BIE is compared with the maximum likelihood estimate (MLE) and a semiparametric estimator (SP) of Engle and Gonzalez-Rivera (1991).

Introduction

Knowledge of the distribution of exchange rates has important implications for theories of international finance and their applications. The specification of the stochastic processes of exchange rates is essential for the options pricing on foreign currencies. Understanding the behavior of exchange rates also helps predict its effects on international trade and investments. In the analysis of capital markets, testing exchange market efficiency requires the information of statistical properties of the exchange rate distribution. Also, the variance of exchange rates is a major risk component in international investing. Furthermore, knowledge of the volatility of exchange rates is important both for portfolio selection and for the evaluation of the performance of international asset portfolios.

The empirical evidence on the distribution of exchange rates, however, has been far from conclusive. While most previous studies have recognized that the rate of change in a foreign currency is not normally distributed, there is a lack of consensus on what type of distribution is most appropriate for describing the behavior of exchange rates. Examples of alternative statistical distributions, which have been commonly suggested in describing the changes in exchange rates, include the symmetric stable Paretian, the Student t, the mixture of normal distributions, and the normal distribution with time-varying parameters (e.g., Friedman and Vandersteel 1982; Booth and Glassman 1987; and Tucker and Scott 1987). Nevertheless, none of these well-documented alternatives has gained general acceptance.

An alternative approach to issues of exchange rates is the ARCH model (see Engle 1982; Bollerslev et al. 1990, for a survey). This model is intuitively appealing because the observed exchange rates seem to exhibit volatility clusters, i.e., periods of high volatility that tend to be

followed by periods of high volatility. Hsieh (1988, 1989a) and Baillie and Bollerslev (1989) applied the ARCH model to daily exchange-rate series, and Diebold and Nerlove (1989) estimated the ARCH model for weekly spot-exchange rates. Overall, the findings of ARCH in exchange rates are important. First, ARCH models are consistent with unconditional leptokurtosis in the changes of exchange rates (e.g., see Westerfield 1977; Boothe and Glassman 1987). Second, ARCH models may prove to be provide particularly helpful tools in future analyses and enhance understanding of currency-option pricing with stochastic volatilities' models (e.g., Hull and White 1987; Melino and Turnbull 1990).

Among all assumptions of ARCH models, a very important one is that the distribution of the disturbance at time t conditional on the available information up to $t-1$ is normal. However, numerous studies (e.g., Hsieh 1988, 1989; Baillie and Bollerslev 1989) showed that the distribution of the changes in exchange rates is, unconditionally as well as conditionally, far from being normal. In fact, leptokurtosis and skewness are frequently present. Hence, the normality assumption seems to be inadequate and often leads to false or inefficient inferences. This is mainly due to the fact that exchange rates are contaminated by some outliers or extreme values so that the conditional distribution looks heavy-tailed.

To account for heavy tails of the conditional distribution, Engle and Bollerslev (1986) and Bollerslev (1987) used student-t rather than normal, since student-t distribution is heavy-tailed relative to the normal distribution. In addition to the students-t distribution, Hsieh (1989b) and Nelson (1991) also used the generalized error distribution (GED), which encompasses the normal, exponential, and uniform distributions. However, Nelson (1991) noted that the GED has only one parameter to control the shape of the conditional distribution, and it may not be flexible enough due to too many outliers in the data.

This paper proposes a BIE that is robust against departure from normality (of the conditional distribution) to describe the behavior of the changes in exchange rates. First, the BIE is used to identify the additive outliers (AO) caused by abnormal information arrivals that may be triggered by changes in domestic policies and international shocks. Identification of outliers allows us to analyze the major economic and political factors that contribute directly to the dramatic changes in exchange rates not described by the model. Second, the performance of the BIE will be compared with the maximum likelihood estimate (MLE), and a semiparametric estimator (SP) (Engle and Gonzalez-Rivera, 1991). Issues related to the assumption of the distribution such as non-normality, leptokurtic, and outlying observations will also be addressed.

The paper is organized as follows. The next section provides some background necessary to understand the proposed BIE and place it in context with related work. Section 3 describes the BIE in details. In Section 4, data and empirical results are reported. Finally, Section 5 provides a summary.

2. Model

Consider an ARCH model suggested by Geweke (1986),

$$y_t | \Psi_{t-1} \sim N(0, \sigma_t^2) \tag{1}$$

$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2,$$

where y_t is the rate of change for the foreign exchange spot rate, Ψ_{t-1} is the information set available at time $t-1$ and σ_t^2 is the conditional variance. Note that the conditional variance σ_t^2 is positive for all values of α . Equation (1) is sometimes referred as the log-ARCH model. The log-likelihood function is

$$\ln L = -\sum_{t=1}^T \left(\log \sigma_t^2 + \frac{y_t^2}{\sigma_t^2} \right) \quad (2)$$

The MLE either maximizes equation (2) or solves the following equation

$$\frac{\partial \ln L}{\partial \theta} = 0, \quad (3)$$

where $\theta = (\alpha_1, \alpha_2)$. Note that if $\alpha_2 = 0$, the changes in exchange rates reduce to a random walk.

Hsieh (1989), Engle and Bollerslev (1986), and Baillie and Bollerslev (1989) have found the MLE of ARCH is sensitive to distributional assumptions. One explanation is that the observations are contaminated by outliers and/or extreme values that make the conditional distribution look heavy tailed. Consequently, the outliers may not be helpful in predicting future variances, and the estimates in the variance function may be unduly influenced by a few extreme observations. These arguments strongly suggest the need of constructing robust-resistant ARCH parameter estimates and use these robust estimates to detect outliers.

Note that equation (1) can be written as

$$\log y_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2 + v_t, \quad (4)$$

where $v_t = \log y_t^2 - \log \sigma_t^2$ are uncorrelated for $t = 1, 2, \dots, T$. Thus equation (1) can be rewritten as an autoregressive model of order 1 (AR(1)) for $\log y_t^2$. Hence, the process $\log y_t^2$ has the same correlation structure as that of an AR(1) process with AR parameter α_2 .

Pantula (1986) recently introduced the following generalized ARCH (GARCH(1, 1)) model that allows the conditional variance to depend not only on past residuals, but also on its own past realizations:

$$\begin{aligned} y_t | \Psi_{t-1} &\sim N(0, \sigma_t^2) \\ \log \sigma_t^2 &= \alpha_1 + \alpha_2 \log y_{t-1}^2 + \alpha_3 \log \sigma_{t-1}^2. \end{aligned} \quad (5)$$

Note that the equation (5) can also be written as

$$\log y_t^2 = \alpha_1 + (\alpha_2 + \alpha_3) \log y_{t-1}^2 + \alpha_3 v_{t-1} + v_t, \quad (6)$$

where $v_t = \log y_t^2 - \log \sigma_t^2$. This reveals that $\log y_t^2$ in equation (5) follows an autoregressive and moving average model (ARMA(1, 1)) with serially uncorrelated v_t .

In the standard ARCH/GARCH, little attention has been given to outlying observations. Jorion (1988) has a model that is very similar to the AO model. In his model, Jorion allows the mean of the exchange rate to follow a jump process, while the variance of the exchange rate follows an ARCH process. However, the present study considers the AO in the ARCH process to the variance but not the mean.

Now we take a more careful look at the outlying observations on ARCH models. Before assessing the effects of outliers on the ARCH models, we define what we mean by outliers in the time series models. Two major types of outliers have been defined by Fox (1972): One is called the additive effects outliers (AO) model; the other is referred as the innovation outlier (IO) model. An IO represents an extraordinary shock at time t influencing y_t, y_{t+1}, \dots , through the dynamic system described by equation (1).

In the IO model, occasional innovations have larger variance than the majority and therefore, can appear as outliers. In the AO model, on the other hand, the isolated outlier has an additive transient character that is unrelated to the time series model. Thus, the AO is also called a gross error, since only the level of t^{th} observation is affected. In fact, the IO outliers transmit their effect through to later observations; AO outliers do not. We also note that IO model will create the heavy-tailed distribution and ARCH model is heavy-tailed. ARCH model, therefore, seems to be able to capture the IO by construction. Assume that the observations are generated from

$$z_t = x_t + e_t \quad (7)$$

where $x_t = \log y_t^2$ follows an AR(1) model in equation (4), and e_t is an independent sequence of variables, independent of the sequence of x_t . The variable e_t has distribution H , given by

$$H = (1 - \varepsilon)\delta_0 + \varepsilon G,$$

where δ_0 is the distribution that assigns probability 1 to the origin and G is an arbitrary distribution. Therefore, with probability $1 - \varepsilon$, the AR(1) process x_t itself is observed, and with probability ε the observation is the AR(1) process x_t plus an error with distribution G . Further insights into the effects of AO to the ARCH model can be seen as follows: Let

$$\begin{aligned} z_t &= x_t + e_t \\ x_t &= \alpha_1 + \alpha_x x_{t-1} + v_t \\ e_t &\sim (1 - \varepsilon)\delta_0 + \varepsilon G. \end{aligned}$$

Making the autoregressive transformation of z_t , we have that

$$z_t - \alpha_2 z_{t-1} = x_t - \alpha_2 x_{t-1} + e_t - \alpha_2 e_{t-1}. \quad (8)$$

Note that the sum of the two uncorrelated moving average (MA(1)) processes on the RHS equation of (8) is MA(1). Hence equation (8) is an ARMA(1, 1) process. That is, the AR(1) model with an AO becomes an ARMA(1, 1) model in equation (8). In other words, the ARCH(1) model with an AO will become a GARCH(1, 1). Hence, GARCH (1, 1) model in equation (5) is able to capture the AO.

Looking at the equation (6) and equation (8), it appears that AO hypothesis implies a testable restriction on the parameters of a GARCH(1,1) model. In particular, the AO hypothesis implies that from equation (8) the estimated AR parameter will be equal to the estimated MA parameter in a GARCH (1,1) model. This AO hypothesis will be tested in a later paper.

3. Bounded Influence Estimation

The foregoing analysis shows that the MLE of the ARCH models may be sensitive to AO-type outliers. Consequently, detection of outlying observations implies that a robust estimation should be used. The motivation for BIE arises from studies such as Krasker and Welsh (1982), Kao and Dutkowsky (1989), and Peracchi (1990a, 1990b, 1991).

The BIE proposed here is an iteratively reweighting technique where the weights decrease as some norms of the score function increases. The BIE for θ , denoted by $\hat{\theta}$ solves

$$\sum_{t=1}^T w(y_t, \hat{\theta}) s(y_t, \hat{\theta}) = 0, \quad (9)$$

where $w(\cdot)$ is a nonnegative weight function, θ is a K by 1 vector of parameters to be estimated and $s(\cdot)$ is the score function such that

$$w(x, \theta) = \min \left\{ 1, \frac{bK^{1/2}}{\left[s^T(x, \theta) A^{-1} s(x, \theta) \right]^{1/2}} \right\}, \quad (10)$$

where

$$A = E \left[w^2(y, \theta) s(y, \theta) s^T(y, \theta) \right]. \quad (11)$$

The influence bound b is specified prior to estimation. Krasker and Welsch (1982) demonstrated that b has lower bound of unity.

The problem of selecting the optimal influence bound has not been conclusively resolved (see Samarov 1985; Powell 1990). Suggested by Krasker et al. (1982), a criterion requires a predetermined level of asymptotic efficiency relative to the MLE at the “ideal” model. Hampel, Rousseeuw, Ronchetti, and Stahel (1986, p. 252) pointed out, however, that such an approach may lead to estimators with very low robustness. They suggested choosing the influence bound near 1. Carroll and Ruppert (1987) and Kao et al. (1989) used these bounds ranged from 1.1 to

1.7 in their empirical studies. Peracchi (1990a) suggested that b is chosen so as to obtaining an average weight of about 95 percent.

Equation (9) implies that the BIE falls within the class of weighted MLE. The BIE modifies the score function and finds the roots of the resulting likelihood functions. Equation (10) describes the choice of observation weights based on a Mahalanobis-type distance of $s(y, \theta)$ from the centroid of $\{s(y, \theta) : t = 1, 2, \dots, T\}$. An observation is downweighting only if its influence exceeds the maximum allowable influence $bK^{1/2}$. Observations with influence below this bound receive a weight of unity. In this way the BIE compares with the MLE while, at the same time, the estimator protects against highly influential observations. From Equation (11) we see that A is a robust version of the second-moment matrix of $s(y, \theta)$.

The influence function (IF) of the BIE is

$$IF(y, \theta) = B^{-1}w(y, \theta)s(y, \theta), \quad (12)$$

where

$$B = -E\{\partial[w(y, \theta)s(y, \theta)]/\partial\theta\}. \quad (13)$$

Note that the influence function (IF) (see, e.g., Hampel, 1986, Peracchi, 1990b) measures the effect, on the asymptotic bias of an estimator, of an arbitrarily small contamination of the assumed statistical model.

The corresponding asymptotic covariance matrix of the BIE, denoted by V , is then

$$V = B^{-1}A B^{-1}. \quad (14)$$

Since the IF is a $K \times 1$ vector, there is no natural ordering for influence. Obtaining a scalar in measuring of influence requires the application of appropriate norm for $IF(y, \theta)$. This norm maps the IF into R^1 , combining the influence of a given observation over each parameter in θ to compute an overall measure of the observation's influence. The Euclidean norm cannot

be used here since it depends heavily upon the scaling of independent variables. A more suitable measure which is independent of the particular parameterization is the self-standardized gross-error sensitivity (e.g., Krasker and Welsch 1982),

$$\lambda = \max \left\{ s^T(y, \theta) A^{-1} s(y, \theta) \right\}^{1/2}. \quad (15)$$

The γ in equation (15) measures the worst effect that a small amount of contamination by gross-error can have on the bias of the BIE. The construction of the weights in equation (15) implies that $\gamma < b$ for suitable choices of the influence bound. Therefore, the foregoing estimator achieves bounded influence. Bounding the gross-error sensitivity ensures robustness, with greater robustness produced by smaller bounds. The details of the computational algorithm can be found in Carroll et al. (1987), Kao et al. (1989), and Peracchi (1990a, 1990b).

Note that bounded influence weights, $w(\bullet)$, provide useful diagnostic information for outliers and influential observations, in particular, and identifying potential sources of model failure. Recently, several nonparametric and semiparametric estimators for the ARCH/GARCH have been discussed in the literature (e.g., Diebold and Nason 1990; Pagan and Ullah 1988; Pagan and Schwert 1990; Robinson 1988). Gallant et al. (1991) used a semi-nonparametric method where the conditional density is estimated with a polynomial expansion using ARCH as a leading term. Engle and Gonzalez-Rivera (1991) estimated the conditional distribution using a nonparametric penalized likelihood density estimation of Tapia and Thompson (1978). Weiss (1986) and Bollerslev and Woodridge (1988) proposed a quasi-maximum likelihood (QMLE) for ARCH and GARCH. These estimators have certain robustness properties (such as consistency), but can be very inefficient, for they disregard entirely the information contained in the parametric assumptions. For example, Engle and Gonzalez-Rivera (1991) showed that the loss of efficiency of the QMLE could go up to 84 percent due to misspecification of the density. The BIE, on the

other hand, provides a compromise between efficiency and robustness, since they take parametric assumptions into account.

4. Data and Empirical Results

The data set consists of daily spot rates of foreign exchange rates (in terms of U.S. dollar) from the International Financial Statistics. Five major currencies are selected: the British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), Japanese Yen (JY), and Swiss Franc (SF). There are 1579 daily observations from May, 1 1980 to June 16, 1986. The analyzed series for each of the United States exchange rate is the first differences of the logarithms of the spot price of a specific currency in terms of dollars. Hence, the data represent the continuously compounded percentage rate of return for holding the particular currency one day.

Table 1 reports the MLE and BIE of the parameters of the ARCH(1) processes. A ZXMIN subroutine of the IMSL libraries is used to compute the maximum likelihood estimators. The algorithm of computing BIE is written in FORTRAN and (9) is solved by subroutine ZSPOW in the IMSL libraries. For a given currency, the first row and second row display the parameter estimates of ARCH(1) process. Standard errors appear in the parentheses. The MLE of α_1 and α_2 are significantly different from zero. The only exception is the estimate of α_2 for the Swiss Franc.

As mentioned earlier, there are some outliers in the daily exchange rate data that may not be representative of the true exchange rate process. Including these data that are not representative may cause bias in the parameter estimation. To assess the effect of outlying observations on the parameter estimates, the ARCH(1) process is re-estimated with the BIE. Column three to column seven in Table 1 report the estimates of the BIE for the ARCH(1) process. Different values of bounds are set (1.1 to 1.7) in the estimation. The smaller the

bounds, the more the data were downweighted. Table 1 shows that parameter estimates are very sensitive to the outliers. In particular, the estimates of α_2 increased about 200 percent to 400 percent for the BIE with the bound equal to 1.7 compared to the MLE. The signs of α_2 for DM, JY, BP and SF changed from negative to positive under the BIE. If the BIE represents the true parameter estimates of the population, then the volatility of exchange rates has been underestimated by a substantial amount when the MLE is used as in most of previous studies.

MLE and BIE for the GARCH(1, 1) model are given in the Table 2. Their results show that using BIE-ARCH(1) lead to significant differences with respect to ARCH(1). This is due to the fact that the BIE is less sensitive than the MLE to local violations of the model assumptions. BIE-GARCH(1,1) and GARCH(1,1) tend to be close.

The BIE-GARCH(1, 1) identified two groups of abnormal data in the foreign exchange rates. The first group includes the “shocks” that cannot be explained by the ARCH(1) and GARCH(1, 1) process. As shown in Table 3 and Table 4, these are large fluctuations in foreign exchange associated with important political and economic events. The second group includes the AO-type outliers that are captured by the GARCH(1, 1) process. The procedure of identifying these AO outliers is as follows. Using the BIE we fitted the exchange rate data to the BIE-ARCH(1) and the BIE-GARCH(1, 1) process. We found some observations are downweighted substantially for the ARCH(1) process but are either not downweighted or just downweighted slightly for the GARCH(1, 1). This means that these observations do not fit the ARCH(1) process well but fit fairly well to the GARCH(1, 1) process. Since the only difference between these two models is that the GARCH(1, 1) process includes a moving average component, these observations must be associated with the AO-type outliers. In this way, we identify the AO effects of economic and political changes that cause the jumps in exchange rate movements.

Table 2 also presents the results obtained by estimating the various currencies using the semiparametric GARCH proposed by Engle and Gonzalez-Rivera (1990) (see Engle and Gonzalez-Rivera for details on the computations). These results show that using semiparametric GARCH does not lead to significant differences with respect to MLE. Semiparametric and MLE estimates tend to be close. It seems that EG's semiparametric ARCH is not robust with respect to outliers, which is not surprising (see Huber, 1981, p. 6). For example, the sample mean is a nonparametric estimator of the population mean, but the sample mean is highly sensitive to outliers and therefore very non-robust.

Table 3 reports the data points that were substantially downweighted by the BIE for the British Pound for the purpose of demonstration. As shown in the table, most of the observations downweighted in the BIE-GARCH(1, 1) process are also downweighted in the BIE-ARCH(1) process. The AO-type outliers are listed in Table 3. The unexplained outliers in Table 3 may be due to the level-shift (LS) type outliers or structural change in Lastrapes (1989), Diebold and Pauly (1988), Chen and Tiao (1990) and Lamoureux and Lastrapes (1990). Further work is needed for explaining the ARCH or GARCH with LS-type outliers (e.g., Gouriéroux and Monfort 1990; Chu, 1991).

Tables 4 documents some major events occurring on these dates identified in Table 3. The events displayed in Tables 4 reflects major policy changes and international turbulence. The findings indicate that these events led to abnormal jumps or fluctuations in the foreign exchange rates for the British Pound.

5. Conclusion

This paper extends the current literature on the distribution of exchange rate changes in a number of ways. First, the parameters of the distribution were estimated with a BIE. The results

of section 4 show that exchange rate changes estimated from the same set of data can differ significantly depending on the choice of the model and estimation technique. In particular, the ARCH(1) can differ significantly from BIE as a consequence of the presence of only a small fraction of extreme observations. This estimation procedure offers an efficient mechanism to downweight outlying observation and therefore, provides more accurate estimates for the parameters of the exchange rate changes distribution. Second, the major political and economic events that caused jumps and abnormal fluctuations in exchange rates were identified by examining the data points that were detected by the BIE. The effects of policy changes and international events on exchange rate movements were carefully analyzed. The analysis provides policy makers very valuable information on the sensitivity of exchange rate to policy shifts and economic events.

Table 1. MLE and BIE for the log-ARCH Process^a

$$y_t | y_{t-1} \sim N(0, \sigma_t^2)$$

$$\log \sigma_t^2 = \alpha_1 + \alpha_2 \log y_{t-1}^2$$

Currency	MLE	BIE (1.7)	BIE (1.5)	BIE (1.3)	BIE(1.1)
Canadian Dollar					
α_1	-11.183 (0.073)	-10.674 (0.068)	-10.653 (0.063)	-10.722 (0.061)	-10.926 (0.057)
$\alpha_2 \times 10^2$	5.932 (0.527)	13.936 (0.421)	15.483 (0.376)	15.438 (0.370)	15.277 (0.344)
Deutsche Mark					
α_1	-9.606 (0.045)	-9.595 (0.057)	-9.648 (0.057)	-9.736 (0.056)	-10.048 (0.049)
$\alpha_2 \times 10^2$	-0.606 (0.362)	4.731 (0.438)	4.748 (0.436)	4.708 (0.433)	5.064 (0.372)
Japanese Yen					
α_1	-10.527 (0.059)	-9.821 (0.052)	-9.861 (0.052)	-9.997 (0.051)	-10.297 (0.049)
$\alpha_2 \times 10^2$	-4.341 (0.401)	7.005 (0.309)	6.993 (0.309)	0.823 (0.306)	6.103 (0.300)
British Pound					
α_1	-9.123 (0.067)	-8.861 (0.061)	-8.809 (0.055)	-8.846 (0.054)	-8.932 (0.053)
$\alpha_2 \times 10^2$	-5.036 (0.571)	12.098 (0.433)	14.213 (0.374)	14.348 (0.366)	14.407 (0.355)
Swiss Franc					
α_1	-8.139 (0.014)	-9.430 (0.021)	-9.497 (0.021)	-9.660 (0.021)	-9.915 (0.019)
$\alpha_2 \times 10^2$	-0.000 (0.130)	4.938 (0.176)	4.890 (0.176)	4.662 (0.174)	4.871 (0.017)

^aAsymptotic standard errors are in parentheses below each coefficient. BIE(1.5) is the BIE with bound to be 1.5. Sample period is from May 1, 1980 to June 16, 1986.

Table 2. MLE, BIE, and SP for the log-GARCH Process

Currency	MLE	BIE (1.7)	BIE (1.5)	BIE (1.3)	BIE(1.1)
Canadian Dollar					
α_1	-0.412 (0.051)	-0.096 (0.021)	-0.071 (0.020)	-0.085 (0.019)	-0.351 (0.018)
$\alpha_2 \times 10^2$	4.038 (0.202)	3.614 (0.194)	3.553 (0.187)	3.667 (0.178)	3.212 (0.069)
α_3	0.919 (0.005)	0.955 (0.003)	0.956 (0.002)	0.954 (0.002)	0.938 (0.002)
Log-likelihood	-8758.823				-7404.278
Deutsche Mark					
α_1	-0.081 (0.031)	-0.263 (0.056)	-0.273 (0.056)	-0.326 (0.059)	-0.141 (0.045)
$\alpha_2 \times 10^2$	1.958 (0.185)	3.174 (0.250)	3.203 (0.248)	3.276 (0.242)	1.732 (0.084)
α_3	0.968 (0.004)	0.938 (0.007)	0.937 (0.007)	0.932 (0.007)	0.963 (0.002)
Log-likelihood	-6768.853				-5448.361
Japanese Yen					
α_1	-0.087 (0.037)	-0.002 (0.007)	-0.001 (0.006)	-0.004 (0.006)	-0.076 (0.004)
$\alpha_2 \times 10^2$	0.534 (0.132)	1.207 (0.117)	1.212 (0.109)	1.253 (0.103)	0.686 (0.093)
α_3	0.985 (0.005)	0.986 (0.002)	0.987 (0.001)	0.987 (0.001)	0.984 (0.002)
Log-likelihood	-7102.765				-5748.399
British Pound					
α_1	-0.053 (0.019)	-0.076 (0.026)	-0.078 (0.025)	-0.082 (0.025)	-0.066 (0.057)
$\alpha_2 \times 10^2$	1.755 (0.172)	3.414 (0.242)	3.527 (0.235)	3.544 (0.229)	1.574 (0.070)
α_3	0.974 (0.003)	0.954 (0.004)	0.953 (0.004)	0.953 (0.004)	0.974 (0.004)
Log-likelihood	-6925.577				-5543.019
Swiss Franc					
α_1	0.245 (0.003)	-0.254 (0.030)	-0.281 (0.030)	-0.312 (0.031)	0.258 (0.002)
$\alpha_2 \times 10^2$	4.193 (0.089)	3.749 (0.146)	4.146 (0.145)	4.369 (0.141)	6.315 (0.091)
α_3	0.974 (0.001)	0.932 (0.005)	0.926 (0.005)	0.922 (0.004)	0.972 (0.002)
Log-likelihood	-6183.429				-4783.725

**Table 3. Selected Downweighted Cases from the BIE(1.7):
The Case of British Pound**

Case Number	Date	BIE-ARCH Weights	BIE-GARCH Weights	Outliers Type
3	5/06/80	.05 ^a	1.00	AO ^b
121	10/21/80	.08	.07	
217	3/10/81	.03	.97	AO
221	3/16/81	.09	1.00	AO
257	5/06/81	.04	1.00	AO
263	5/14/81	.04	.03	
512	5/03/82	.05	.15	
587	8/18/82	.06	.61	AO
596	8/31/82	.02	.38	AO
691	1/11/83	.02	.02	
751	4/05/83	.02	.06	
771	5/03/83	.04	.57	AO
811	6/28/83	.05	.61	AO
845	8/15/83	.08	.69	AO
945	1/03/84	.05	.14	
1035	5/08/83	.02	.04	
1161	10/31/84	.09	1.00	AO
1244	2/27/85	.04	.07	
1246	3/01/85	.01	.49	AO
1258	3/19/85	.05	.09	
1264	3/27/85	.02	.16	
1283	4/24/85	.09	.16	
1307	5/28/85	.04	.04	
1355	8/02/86	.09	.13	
1361	8/12/85	.02	.49	AO
1380	9/06/85	.03	.06	
1391	8/23/85	.03	.04	
1534	4/16/86	.09	.51	AO

^aCases are selected when weights are less than .10.

^bAO-type outliers are indicated when weights in BIE-GARCH are large (say ,.40)

**Table 4. Important Events Coincide with the Shocks Found by the BIE:
The Case of British Pound**

Date	Events
5/14/81	Agreement reached on the increase in international minimum interests rates.
6/4/81	Mexico reduced its oil price on June 3.
1/11/83	Clearing banks raised base lending rate from 10 to 11 percent on January 11.
2/27/85	On this day there was coordinated central bank frozen exchange intervention to restrain the dollar.
3/19/85	Market anticipated a fall in base rates of as much as 1 percent.
3/27/85	National Westminster and Lloyds banks prepared to announce that they would be cutting their based rates to 13 percent.
4/24/85 to 4/29/85	Rumors of a \$1 cut in the price of Soviet crude oil occurred.
8/2/85	There was severe concern in the foreign exchange market that interest rates might be pushed down.
9/6/85 to 9/9/85	Break with OPEC pricing by Saudi Arabia.
9/23/85 to 9/27/85	Meeting of the G5 (group of five industrial countries) in New York on weekend of September 21 st and 23 rd .

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